Observer-Based Stabilization of T–S Fuzzy Systems With Input Delay
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Abstract—This paper discusses the stabilization of Takagi–Sugeno (T–S) fuzzy systems with bounded and time-varying input delay. The robust stabilization via state feedback is first addressed, and delay-dependent stabilization conditions are proposed in terms of LMIs. Observer-based feedback stabilization is also discussed for T–S fuzzy input delay systems without uncertainties. A separate design principle is developed. Some illustrative examples are given to show the effectiveness and feasibility of the proposed methods.

Index Terms—Delay-dependent, fuzzy control, input delay, observer, Takagi–Sugeno (T–S) fuzzy system.

I. INTRODUCTION

SINCE the first paper on the Takagi–Sugeno (T–S) model [17] was published, the T–S fuzzy model has attracted a great attention and many systematic fuzzy controller design methods (via state feedback and output feedback) have been developed based on the T–S model. For a detailed review, see [5], [13], [18], [20], [22], [27], and the references therein. Recently, in view of possible time delays in practical systems, stability analysis and control synthesis of T–S fuzzy systems with state delay have been also addressed. The problems of stabilization and $H_\infty$ control have been studied based on delay-independent method in [2], [27], and [10]. More recently, some new delay-dependent fuzzy controller design approaches have been also developed by [6] and [11]. The problem of stabilization and synthesis of T–S fuzzy time-delay descriptor systems has been considered in [12], while the robust stabilization approaches for T–S fuzzy systems with interval time delay can be found in [21] and [7]; for more details, see [4]. However, it should be pointed out that all the aforementioned results were obtained for systems with no input delays. Note that, in modern industrial systems, sensors, controllers, and plants are often connected over a network medium [3]. Therefore, since the sampling data and controller signals are transmitted through a network, network-induced delays and data dropout in networked control systems are always inevitable. So, it is important both in theory and practice to consider the input delay when controllers are designed. So far, some controller design methods have been proposed for linear systems with input delay. In [8] and [14], memoryless controllers have been designed by using Razumikhin methods, and in [15], [25], and [26], memory controllers have been developed by using Lyapunov–Krasovskii functional approach. However, little attention has been paid to the problem of stabilization of T–S fuzzy systems with input delays.

In this paper, we will design a fuzzy controller to achieve the asymptotic stability of the resulting closed-loop system with input delay. The input delay is assumed to be bounded and time-varying. Based on Lyapunov–Krasovskii method, a robust state feedback controller is designed for an uncertain T–S fuzzy system with input delay. Then, observer-based feedback stabilization is addressed for T–S fuzzy input delay systems. To apply a separate design principle, it is assumed that the systems in consideration do not contain uncertainties. All results are presented in terms of LMIs. Finally, two examples are used to illustrate the effectiveness and feasibility of our approaches.

Throughout this paper, an identity matrix, of appropriate dimensions, will be denoted by $I$. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n \times n}$ means that the matrix $X$ is real symmetric positive definite (respectively, positive semi-definite). If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol “$\in$” in a matrix $A \in \mathbb{R}^{n \times n}$ stands for the transposed elements in the symmetric positions.

II. PROBLEM STATEMENTS

TS fuzzy dynamic model with input delay is described by fuzzy IF THEN rules, which represent local linear input-output relations of nonlinear systems. Consider an uncertain nonlinear system which is described by the following T–S fuzzy model:

**Plant Rule $i$:**

\[
\begin{align*}
&\text{IF } \theta_1(t) \text{ is } N_{i1} \text{ and } \cdots \text{ and } \theta_p(t) \text{ is } N_{ip}, \text{ THEN} \\
&x(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t - \tau(t)), \\
y(t) = C_i x(t), \\
x(0) = x_0, \quad u(t) = \varphi(t), \quad t \in [-\sigma, 0], \quad i = 1, 2, \ldots, k
\end{align*}
\]

where $N_{ij}$ are the fuzzy sets, $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ and $C_i \in \mathbb{R}^{p \times n}$ are constant matrices. Scalar $k$ is the number of IF-Then rules. $\theta_1(t), \theta_2(t), \ldots, \theta_p(t)$ are the premise variables.
It is assumed that the premise variables do not depend on the input $u(t)$. Function $\tau(t) > 0$ is an unknown, bounded and time-varying delay, and the constant $\sigma > 0$ is a known upper bound of $\tau(t)$. The matrices $\Delta A_i$ and $\Delta B_i$ denote the uncertainties in the system and take the form of

$$ \begin{align*} 
[\Delta A_i, \Delta B_i] & = MF(t)[E_i, E_i g]
\end{align*} $$

where $M$, $E_i$, and $E_i g$ are known constant matrices and $F(t)$ is an unknown matrix function with the property $F^T(t)F(t) \leq I$.

Given a pair of $(x(t), u(t))$, the final output of the fuzzy system is inferred as follows:

$$ \begin{align*} 
\dot{x}(t) & = \sum_{i=1}^{k} h_i(\theta(t)) [(A_i + \Delta A_i)x(t) \\
& \quad + (B_i + \Delta B_i)u(t - \tau(t))] \\
x(0) & = x_0, \quad u(t) = \varphi(t), \quad t \in [-\sigma, 0]
\end{align*} $$

(1)

where $h_i(\theta(t)) = \mu_i(\theta(t))/\sum_{j=1}^{k} \mu_j(\theta(t))$, $\mu_i(\theta(t)) = \prod_{j=1}^{i} N_{ij}(\theta_j(t))$, and $N_{ij}(\theta_j(t))$ is the degree of the membership of $\theta_j(t)$ in $N_{ij}$. In this paper, we assume that $\mu_i(\theta(t)) \geq 0$ for $i = 1, 2, \ldots, k$ and $\sum_{i=1}^{k} \mu_i(\theta(t)) > 0$ for all $t$. Therefore, $h_i(\theta(t)) \geq 0$ for $i = 1, 2, \ldots, k$, and $\sum_{i=1}^{k} h_i(\theta(t)) = 1$.

For system (1), based on the parallel distributed compensation, the following fuzzy control law is employed to deal with the problem of robust stabilization:

**Controller Rule i:**

IF $\theta_i(t)$ is $N_{i1}$ and $\ldots$ $\theta_i(t)$ is $N_{ip}$ THEN

$$ u(t) = K_i x(t), \quad i = 1, 2, \ldots, k. $$

(2)

Hence, the overall fuzzy control law is represented by

$$ u(t) = \sum_{i=1}^{k} h_i(\theta_i(t)) K_i x(t) $$

(3)

where $K_i$ ($i = 1, 2, \ldots, k$) are the local control gains.

Note that $u(t - \tau(t)) = \sum_{i=1}^{k} h_i(\theta(t - \tau(t))) K_i x(t - \tau(t))$. Therefore, it is natural and necessary to assume that the functions $h_i(\theta_i(t)) (i = 1, 2, \ldots, k)$ are well defined for all $t \in [-\sigma, 0]$, and satisfy the following properties:

$$ h_i(\theta_i(t - \tau(t)) \geq 0 (i = 1, 2, \ldots, k), \quad \sum_{i=1}^{k} h_i(\theta_i(t - \tau(t)) = 1. $$

Before stating our main results, we first give the following Lemmas, which will be used to prove our main results.

**Lemma 1:** [15] Let $a \in R^n$, $b \in R^m$, and $N \in R^{m \times m}$. Then, for any matrices $X$, $Y$, and $Z$ with appropriate dimensions, the following inequality holds:

$$ -2x^T N b \leq \begin{bmatrix} [a] & [X] \\ [b] & [Y - N] \\ [a] & [Z] \end{bmatrix} \begin{bmatrix} b \end{bmatrix} $$

where $X$, $Y$, and $Z$ satisfy

$$ \begin{bmatrix} X & Y \\ & & & Z \end{bmatrix} \preceq 0. $$

**Lemma 2:** [16] For any constant matrix $M > 0$, any scalars $a$ and $b$ with $a < b$, and a vector function $x(t) : [a, b] \rightarrow R^n$ such that the integrals concerned are well defined

$$ \begin{bmatrix} \int_a^b x(s) ds \\ \int_a^b x(s) ds \end{bmatrix}^T M \begin{bmatrix} \int_a^b x(s) ds \\ \int_a^b x(s) ds \end{bmatrix} \leq (b-a)^T \int_a^b x^T(s) M x(s) ds \int_a^b x(s) ds.$$

**Lemma 3:** [23] Let $M$, $E$, and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t)F(t) \leq I$. Then, the following inequality holds for any constant $\varepsilon > 0$

$$ MF(t) E + E^T F^T(t) F(t) M^T \preceq \varepsilon MMT + \varepsilon^{-1} E^T E. $$

III. MAIN RESULTS

In this section, we will first study the robust stabilization of the system (1), and then develop an observer-based feedback design scheme.

**A. State Feedback Stabilization**

For convenience, let $\bar{A}_i = A_i + \Delta A_i$, $\bar{B}_i = B_i + \Delta B_i$, $x(\tau) = x(t - \tau(t))$, $h_i = h_i(\theta_i(t))$ and $h_i(\tau) = h_i(\theta_i(t - \tau(t))$.

Then, associated with the control law (3), the system (1) can be expressed as follows:

$$ \begin{align*} 
\dot{x}(t) & = \sum_{i=1}^{k} h_i(\theta_i(t)) [(A_i + \Delta A_i)x(t) \\
& \quad + (B_i + \Delta B_i)u(t - \tau(t))] \\
& = \sum_{i,j=1}^{k} h_i h_j(\tau) \left( \bar{A}_i x(t) + \bar{B}_i K_j x(\tau) \right). 
\end{align*} $$

(4)

Note that $x(t) - x(\tau) = \int_{t-\tau}^{t} \dot{x}(s) ds$. Then, (4) can be rewritten as

$$ \dot{x} = \sum_{i,j=1}^{k} \int_{t-\tau}^{t} \dot{x}(s) ds. $$

(5)

Thus, for the robust stabilization problem of system (1), we have the following result.

**Theorem 1:** Given scalars $\sigma > 0$ and $d_1 > 0$, the system (5) is asymptotically stable for any $\tau(t)$ satisfying $0 \leq \tau(t) \leq \sigma$ if there exist matrices $X > 0$, $H > 0$, $X_{1i}$, $Y_{1i}$, $Z_{1i}$, $X_{2i}$, $Y_{2i}$, $Z_{2i}$, and $N_i$ as well as constants $\varepsilon_{i,j} > 0$, so that for $i, j = 1, 2, \ldots, k$ [see (6)–(8), shown at the bottom of the next page], where $\Phi_i = A_i X + X A_i^T + B_i N_i + N_i^T B_i^T + X_{1i} + \varepsilon_{i,j} M M T$. Moreover, the feedback gain matrices $K_i$ ($i = 1, 2, \ldots, k$) are given by

$$ K_i = N_i X^{-1}. $$

**Proof:** Pre- and postmultiplying (6) by $\text{diag}[P P P I]$ with $P = X^{-1}$ and then applying Schur complement yield
(9), shown at the bottom of the page, where \( \Phi_i = P(A_i + B_i K_j) + (A_i + B_i K_j)^T P + X_{i1} + \epsilon_{ij} P M M T P + \epsilon_{ij}^{-1}(E_i + E_{ij}^T E_i + E_{ij}^T \mu P X_{i1} \mu P E_{ij}^T) \), \( P_1 = d_i P \), \( T = PHP \), \( X_{i1} = P X_{i2} \mu P \), \( Y_{ij} = P Y_{ij} \mu P \), \( \tilde{Z}_{ij} = P \tilde{Z}_{ij} \mu P \). Multiply both sides of (7) and (8) by \( \text{diag}[P P] \) to obtain the equivalent inequalities, as follows:

\[
\begin{bmatrix}
X_{i1} & Y_{i1} \\
* & \tilde{Z}_{i1}
\end{bmatrix} \geq 0 \quad (10)
\]

\[
\begin{bmatrix}
X_{i2} & Y_{i2} \\
* & \tilde{Z}_{i2}
\end{bmatrix} \geq 0. \quad (11)
\]

Now take \( V = V_1 + V_2 \) as a Lyapunov function candidate with

\[
V_1 = z^T(t) P z(t)
\]

\[
V_2 = \int_{t-\tau}^t (s-t+\sigma) \dot{z}(t) \dot{z}(s) ds. \quad (12)
\]

Differentiating \( V_1 \) along the trajectory of (5) produces

\[
\dot{V}_1 = \sum_{i=1}^k h_i h_j(\tau) x^T [P(A_i + B_i K_j) + (A_i + B_i K_j)^T P] x
\]

\[
- \sum_{i=1}^k h_i h_j(\tau) 2x^T P B_i K_j \int_{t-\tau}^t \dot{z}(s) ds. \quad (13)
\]

According to Lemmas 1 and 2, it follows from (10) that

\[
- \sum_{i=1}^k h_i h_j(\tau) 2x^T P B_i K_j \int_{t-\tau}^t \dot{z}(s) ds \leq \sum_{i=1}^k h_i h_j(\tau)
\]

\[
\times \left[ \int_{t-\tau}^t x^T \right] \begin{bmatrix}
X_{i1} & Y_{i1} - P B_i K_j \\
* & \tilde{Z}_{i1}
\end{bmatrix} \begin{bmatrix}
\dot{X}_{i2} & \dot{Y}_{i2} \\
* & \dot{\tilde{Z}}_{i2}
\end{bmatrix}. \quad (14)
\]

Furthermore, substituting (14) into (13) gives

\[
\dot{V}_1 \leq \sum_{i=1}^k h_i h_j(\tau) x^T [P(A_i + B_i K_j) + (A_i + B_i K_j)^T P] x
\]

\[
+ \sum_{i=1}^k h_i h_j(\tau) \left[ \int_{t-\tau}^t \dot{z}(s) ds \right]^T \begin{bmatrix}
X_{i1} & Y_{i1} - P B_i K_j \\
* & \tilde{Z}_{i1}
\end{bmatrix} \begin{bmatrix}
\dot{X}_{i2} & \dot{Y}_{i2} \\
* & \dot{\tilde{Z}}_{i2}
\end{bmatrix}. \quad (15)
\]

In addition, it can be easily verified by using Lemma 2 that

\[
\dot{V}_2 = \sigma \dot{x}^T T \dot{x} - \int_{t-\tau}^t \dot{x}^T T \dot{x} ds
\]

\[
\leq \sigma \dot{x}^T T \dot{x} - \int_{t-\tau}^t \dot{x}^T T \dot{x} ds
\]

\[
\leq \sigma \dot{x}^T T \dot{x} - \frac{1}{\sigma} \left[ \int_{t-\tau}^t \dot{x} ds \right]^T T \left[ \int_{t-\tau}^t \dot{x} ds \right]. \quad (16)
\]

where the inequalities \( T > 0 \) and \( 0 \leq \tau(t) \leq \sigma \) are used. Thus, by combining (15) and (16) together, one gets

\[
\dot{V} \leq \sum_{i=1}^k h_i h_j(\tau) x^T [P(A_i + B_i K_j)
\]

\[
(A_i + B_i K_j)^T P + X_{i1}] x
\]

\[
+ \sum_{i=1}^k h_i h_j(\tau) 2x^T P B_i K_j \int_{t-\tau}^t \dot{z}(s) ds + \sigma \dot{x}^T T \dot{x}
\]

\[
+ \sum_{i=1}^k \left[ \int_{t-\tau}^t \dot{x} ds \right]^T \begin{bmatrix}
Z_{i1} - \frac{1}{\sigma} T \end{bmatrix} \left[ \int_{t-\tau}^t \dot{x} ds \right]. \quad (17)
\]

\[
\Phi_i = d_i X A_i^T + d_i N_i^T B_i^T + \epsilon_{ij} d_i M M T
\]

\[
X_{i2} + \sigma H - 2d_i X + \epsilon_{ij} d_i^2 R M M T
\]

\[
* \quad *
\]

\[
Y_{i1} - B_i N_j \quad Y_{i2} - d_i B_i N_j
\]

\[
X_{i1} + Z_{i1} - \frac{1}{\sigma} H \quad 0
\]

\[
* \quad *
\]

\[
\begin{bmatrix}
X_{i1} & Y_{i1} \\
* & Z_{i1}
\end{bmatrix} \geq 0
\]

\[
\begin{bmatrix}
X_{i2} & Y_{i2} \\
* & Z_{i2}
\end{bmatrix} \geq 0
\]

\[
\Theta = \begin{bmatrix}
\Phi_i & (A_i + B_i K_j)^T P_1 + \epsilon_{ij} P M M T P_1
\quad Y_{i1} - B_i N_j \quad X E_i^T + N_i^T E_i^T
\quad Y_{i2} - d_i B_i N_j \quad 0
\quad Z_{i1} + Z_{i2} - \frac{1}{\sigma} H \quad N_i^T E_i^T
\quad * \quad * \quad * \quad -\epsilon_{ij} M
\end{bmatrix} < 0 \quad (6)
\]

\[
\begin{bmatrix}
X_{i1} & Y_{i1} \\
* & Z_{i1}
\end{bmatrix} \geq 0
\]

\[
\begin{bmatrix}
X_{i2} & Y_{i2} \\
* & Z_{i2}
\end{bmatrix} \geq 0
\]

\[
\Theta = \begin{bmatrix}
\Phi_i & (A_i + B_i K_j)^T P_1 + \epsilon_{ij} P M M T P_1
\quad Y_{i1} - B_i N_j \quad Y_{i2} - B_i K_j \quad \tilde{Y}_{i1} - B_i K_j - \epsilon_{ij}^{-1}(E_i + E_{ij})^T E_{ij} K_j
\quad \tilde{Y}_{i2} - B_i K_j \quad \tilde{Z}_{i1} + \tilde{Z}_{i2} + \epsilon_{ij}^{-1}(E_{ij} K_j)^T E_{ij} K_j - \frac{1}{h_k} T
\end{bmatrix} < 0 \quad (9)
\]
It is clearly seen that

\[ 0 = 2 \dot{t}^T P_1 \dot{x} - 2 \dot{t}^T P_3 \dot{x} \]
\[ = 2 \sum_{i,j=1}^{k} h_i h_j(\tau) \left[ (\bar{A}_i + B_i K_j)x - B_i K_j \int_{t-\tau}^{t} \dot{x}(s) ds \right]^T P_1 \dot{x} \]
\[ - 2 \dot{t}^T P_3 \dot{x} \]
\[ = \sum_{i,j=1}^{k} h_i h_j(\tau) \dot{t}^T (\bar{A}_i + B_i K_j)^T P_1 \dot{x} - 2 \dot{t}^T P_3 \dot{x} \]
\[ - \sum_{i,j=1}^{k} h_i h_j(\tau) \dot{t}^T P_1 \dot{B}_i K_j \int_{t-\tau}^{t} \dot{x}(s) ds. \]  

(18)

Similar to (14), by using (11) and Lemmas 1 and 2, it follows that

\[ - \sum_{i,j=1}^{k} h_i h_j(\tau) \dot{t}^T P_1 \dot{B}_i K_j \int_{t-\tau}^{t} \dot{x}(s) ds \leq \sum_{i=1}^{k} h_i h_j(\tau) \int_{t-\tau}^{t} \dot{x}(s) ds \]
\[ \times \left[ \dot{x}(t-\tau) \int_{t-\tau}^{t} \dot{x}(s) ds \right]^T \left[ \ddot{x}_{i2} \ddot{Y}_{i2} - P_1 \dot{B}_i K_j \right] \left[ \int_{t-\tau}^{t} \ddot{x}(s) ds \right]. \]  

(19)

Then, (18) and (19) imply that

\[ 0 \leq \sum_{i,j=1}^{k} h_i h_j(\tau) \dot{t}^T (\bar{A}_i + B_i K_j)^T P_1 \dot{x} - 2 \dot{t}^T P_3 \dot{x} \]
\[ + \sum_{i=1}^{k} h_i h_j(\tau) \int_{t-\tau}^{t} \dot{x}(s) ds \]
\[ \times \left[ \ddot{x}_{i2} \ddot{Y}_{i2} - P_1 \dot{B}_i K_j \right] \left[ \int_{t-\tau}^{t} \ddot{x}(s) ds \right]. \]  

(20)

Now, define \( z^T = [\dot{z}^T \ddot{z}^T \int_{t-\tau}^{t} \dot{z}^T ds] \). Then, put (17) and (20) together to get the following inequality:

\[ \dot{V} \leq \sum_{i,j=1}^{k} h_i h_j(\tau) z^T \]
\[ \times \left[ \Psi_i \left( \bar{A}_i + B_i K_j \right)^T P_1 \Sigma_i - P_1 \dot{B}_i K_j \right] \left[ \int_{t-\tau}^{t} \ddot{x}(s) ds \right]. \]  

(21)

\[ \Delta \Pi \leq \varepsilon_{ij} \left[ \begin{array}{ccc} P M_{i} M_{j}^T P & 0 \\ P_{11} M_{i} M_{j}^T P & 0 \end{array} \right] \varepsilon_{ij}^{-1} \left[ \begin{array}{ccc} (E_i + E_{db} K_j)^T (E_i + E_{db} K_j) & 0 \\ 0 & (E_i + E_{db} K_j)^T E_{db} K_j \end{array} \right] \]  

(22)
system in spite of the appearance of an input-delay, which is smaller than or equal to $\sigma$.

### B. Observer-Based Stabilization

Although a design approach of fuzzy control is developed in the previous subsection, its main drawback comes from the requirement for predetermining all the state variables of systems. However, in the real control problem, not all of the state variables are available. Therefore, it is necessary to design an output feedback control law. In this subsection, we consider observer-based output feedback stabilization for system (1). To develop a separation design principle, we consider the special case that the system (1) does not contain uncertainties, i.e., $\Delta A_i = 0$ and $\Delta B_i = 0$. The following observer rules are employed:

**Observer Rule** $\hat{i}$:

If $\theta_i(t)$ is $N_{i1}$ and $\cdots \theta_p(t)$ is $N_{ip}$ THEN

$$\begin{align*}
\dot{\hat{x}} &= A_i\hat{x} + B_i u(\tau) + L_i(y - \hat{y}) \\
\dot{\hat{y}} &= C_i \hat{x}, \quad i = 1, 2, \ldots, k
\end{align*}$$

where $L_i \in R^{n \times q}$ is a constant observer gain matrix to be determined.

The defuzzified output of (24) is represented as follows:

$$\begin{align*}
\dot{x} &= \sum_{i=1}^{k} h_i(\theta(t)) (A_i \hat{x} + B_i u(\tau) + L_i(y - \hat{y})) \\
\dot{y} &= \sum_{i=1}^{k} h_i(\theta(t)) C_i \hat{x}.
\end{align*}$$

Let $e = x - \hat{x}$ denote the observation error. Then the error dynamic can be expressed as follows:

$$e = \sum_{i,s=1}^{k} h_i h_s (A_i - L_i C_s) e_s.$$  \hspace{1cm} (26)

Choose a control law as

$$u(t) = \sum_{i=1}^{k} h_i(\theta(t)) K_i \hat{x}$$

and define $\pi^T = [e^T \ e^T]$. Then, associated with the control law (27), the closed-loop system can be expressed as follows:

$$\begin{align*}
\dot{\hat{x}} &= \sum_{i,j,s=1}^{k} h_i h_j(\tau) h_s (\bar{A}_{is} \bar{x} + \bar{B}_i \bar{K}_j \bar{y}(\tau)) \\
&= \sum_{i,j,s=1}^{k} h_i h_j(\tau) h_s (\bar{A}_{is} + \bar{B}_i \bar{K}_j) \bar{x} \\
&- \sum_{i,j,s=1}^{k} h_i h_j(\tau) h_s \bar{B}_i \bar{K}_j \int_{t-\tau}^{t} \bar{y}^T(s) ds
\end{align*}$$

where

$$\bar{A}_{is} = \begin{bmatrix} A_i & 0 \\ 0 & A_i - L_i C_s \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \bar{K}_j = [K_j - K_j].$$

For the observer-based feedback stabilization, we have the following result.

**Theorem 2:** Given scalars $\sigma > 0$, $d_1 > 0$ and $d_2 > 0$, the system (28) is asymptotically stable if there exist some matrices $X > 0$, $Q > 0$, $H > 0$, $G > 0$, $X_{i1}, X_{i2}, Y_{ij}, Z_{i1}, Z_{i2}, F_i$, and $N_i$ satisfying the following LMIs for $i,j,s = 1,2, \ldots, k$

$$\begin{align*}
\Phi_{ij} &= X A_i^T + F_i^T F_i^T + Y_{i11} - B_i F_i \\
\Psi_{is} &= X_i + \sigma H - 2d_i X_i + Y_{i12} - B_i N_i \\
\Psi_{is} &= X_i + \sigma G - 2d_i G
\end{align*}$$

with

$$\begin{align*}
X_{i1} &= \begin{bmatrix} X_{i11} & X_{i12} \\ X_{i12}^T & X_{i13} \end{bmatrix}, \quad Y_{i1} = \begin{bmatrix} Y_{i11} & Y_{i12} \\ 0 & 0 \end{bmatrix} \\
X_{i2} &= \begin{bmatrix} X_{i21} & X_{i22} \\ X_{i22}^T & X_{i23} \end{bmatrix}, \quad Y_{i2} = \begin{bmatrix} Y_{i21} & Y_{i22} \\ 0 & 0 \end{bmatrix} \\
Z_{i1} &= \begin{bmatrix} Z_{i11} & Z_{i12} \\ Z_{i12}^T & Z_{i13} \end{bmatrix}, \quad Z_{i2} = \begin{bmatrix} Z_{i21} & Z_{i22} \\ Z_{i22}^T & Z_{i23} \end{bmatrix}
\end{align*}$$

Moreover, the feedback gain matrices and observer gain matrices are, respectively, given by

$$K_i = F_i X_i^{-1}, \quad L_i = Q_i^{-1} N_i.$$  \hspace{1cm} (31)

**Proof:** First, define matrices as follows:

$$\Xi = \begin{bmatrix} \Phi_{ij} & (A_i + B_i K_j)^T P & \Psi_{is} \end{bmatrix} \begin{bmatrix} Y_{i11} - B_i F_i \\ X_{i12} + \sigma T - 2d_i \end{bmatrix} \begin{bmatrix} Z_{i11} + Y_{i12} - B_i N_i \\ 2d_i \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} X_{i12} - PB_i K_j & 0 & \Psi_{is} \\ -B_i K_j & X_{i22} + \sigma G - 2d_i Q & \Psi_{is} \\ Y_{i11} & Y_{i12} + P_i B_i K_j & Y_{i12} \end{bmatrix} \begin{bmatrix} Y_{i11} - B_i F_i \\ X_{i12} + \sigma T - 2d_i \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \Psi_{is} & (A_i - L_i C_s)^T Q \\ X_{i22} + \sigma G - 2d_i Q \\ Z_{i12} + Z_{i22} - \frac{1}{\sigma} G \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ \frac{1}{\sigma} \end{bmatrix}$$
\[ \bar{X}_{i_1} = P(A_i + B_i K_j) + (A_i + B_i K_j)^T P + \bar{X}_{i_1} \]
\[ \bar{Y}_{i_2} = P X_{i_1} P \]
\[ \bar{Z}_{i_2} = P X_{i_1} P \]

where \( P = X^{-1} \) and \( T = PHP \). Then, pre- and postmultiplying (29) by \( \text{diag}[P P P] \) gives \( \Omega < 0 \). Condition (30) means that \( \Omega < 0 \). Consequently, since \( \Omega < 0 \) and \( \Omega^{-1} < 0 \), there must exist a constant \( \delta \geq 1 \) such that

\[ \Xi - \Gamma \frac{1}{\delta} \Omega^{-1} \Gamma^T < 0 \]

which implies that

\[ \begin{bmatrix} \Xi & \Gamma \\ \ast & \delta \Omega \end{bmatrix} < 0. \]

(34)

In addition, (31) implies that

\[ \begin{bmatrix} X_{i_1} & Y_{i_2} \\ * & Z_{i_2} \end{bmatrix} = \begin{bmatrix} X_{i_1} & X_{i_2} \\ * & Y_{i_1} \end{bmatrix} \begin{bmatrix} X_{i_1} & 0 \\ * & Z_{i_2} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ * & Z_{i_2} \end{bmatrix} \geq 0 \]

from which it is evident that for \( \delta \geq 1 \)

\[ \begin{bmatrix} X_{i_1} & X_{i_2} & Y_{i_1} & Y_{i_2} \\ * & * & 0 & 0 \\ * & * & Z_{i_2} & Z_{i_2} \\ * & * & * & \delta Z_{i_2} \end{bmatrix} \geq 0 \]

(35)

holds. Furthermore, multiplying both sides of (35) by \( \text{diag}[P P P] \) shows that

\[ \bar{X}_{i_1} = P \bar{X}_{i_1} P \]
\[ \bar{Y}_{i_2} = P \bar{Y}_{i_2} P \]
\[ \bar{Z}_{i_2} = P \bar{Z}_{i_2} P \]

with \( \bar{X}_{i_1} = \delta X_{i_2} \) and \( \bar{Z}_{i_2} = \delta Z_{i_2} \).

Similarly, from (32), we can prove that for \( \delta \geq 1 \)

\[ \begin{bmatrix} \bar{X}_{i_1} & \bar{X}_{i_2} \\ * & \bar{Y}_{i_1} \end{bmatrix} \begin{bmatrix} \bar{Y}_{i_2} \\ * \end{bmatrix} = \begin{bmatrix} \bar{X}_{i_1} & \bar{Y}_{i_1} \\ * & \bar{Z}_{i_2} \end{bmatrix} \begin{bmatrix} \bar{Y}_{i_2} \\ * \end{bmatrix} \geq 0 \]

(36)

with \( \bar{X}_{i_1} = \delta X_{i_2} \) and \( \bar{Z}_{i_2} = \delta Z_{i_2} \).

Now, to develop a separate design principle, choose a Lyapunov function candidate as

\[ V = \bar{x}^T P \bar{x} + \int_{t-\sigma}^{t} (s - t + \sigma) \bar{x}^T T \bar{x} ds \]

where

\[ P = \begin{bmatrix} P & 0 \\ 0 & \delta Q \end{bmatrix} \]
\[ T = \begin{bmatrix} T & 0 \\ 0 & \delta G \end{bmatrix} \]

Then, similar to (13) and (16), a simple calculation shows that the derivative of \( V \) along the trajectory of (28) satisfies the following inequality:

\[ \dot{V} \leq \sum_{i,j=1}^{k} h_i h_j (t) h_a 2 \pi T P (\bar{X}_{i_1} + \bar{B}_j K_j) \sigma \]
\[ - \sum_{i,j=1}^{k} h_i h_j (t) h_a 2 \pi T P \bar{B}_j K_j \int_{t-\tau}^{t} \bar{x}^T (s) ds + \sigma \pi T T \sigma \]
\[ - \frac{1}{\sigma} \left( \int_{t-\tau}^{t} \bar{x} ds \right)^T T \left( \int_{t-\tau}^{t} \bar{x} ds \right). \]

(38)

By using Lemma 1 and (36), the following inequality can be verified:

\[ \int_{t-\tau}^{t} \bar{x}^T (s) ds \]
\[ \int_{t-\tau}^{t} \bar{x} ds \]

Similar to (20), by taking \( P_1 = \begin{bmatrix} d_1 P & 0 \\ 0 & d_2 Q \end{bmatrix} \) > 0 with \( d_1 > 0 \) and \( d_2 > 0 \) being constants and using Lemma 1 and (37), we have

\[ 0 \leq \sum_{i,j=1}^{k} h_i h_j (t) h_a 2 \pi T (\bar{X}_{i_1} + \bar{B}_j K_j) \sigma \]
\[ + \sum_{i,j=1}^{k} h_i h_j (t) h_a \left( \int_{t-\tau}^{t} \bar{x}^T (s) ds \right)^T \]
\[ \int_{t-\tau}^{t} \bar{x} ds \]

(39)

Let \( \pi T = \left[ \begin{array}{c} \pi T \\ \pi T \end{array} \right] \int_{t-\tau}^{t} \pi T ds \). Then, from (38)–(40), the following inequality can be verified:

\[ \dot{V} \leq \sum_{i,j=1}^{k} h_i h_j (t) h_a \pi T \]
\[ \int_{t-\tau}^{t} \bar{x} ds \]

(41)

with \( \Theta_{ij} = P (\bar{X}_{i_1} + \bar{B}_j K_j) + (\bar{A}_{i_2} + \bar{B}_j K_j)^T P + \bar{X}_{i_2} \).
Now, define an orthogonal matrix with appropriate dimension $\Sigma(i,j)$ in such a way that premultiplying a matrix $\Pi$ by $\Sigma(i,j)$ is equivalent to exchanging the $i$th row and the $j$th row of $\Pi$ and postmultiplying $\Pi$ by $\Sigma(i,j)$ is equivalent to exchanging the $i$th column and $j$th column. Furthermore, let $\Sigma = \Sigma(3,5) \times \Sigma(3,4) \times \Sigma(2,3)$, $z^T = [x^T \quad \tilde{x}^T \quad \int_0^t \tilde{x}^T \, ds]$, and $\overline{e} = [\overline{e}^T \quad 0]$. Then, it follows from (41) that

$$
\dot{V} \leq \sum_{i,j,s=1}^k h_i h_j h_s \Sigma^{T} \Sigma \Delta \Sigma \Sigma^{T} \Sigma \\
= \sum_{i,j,s=1}^k h_i h_j h_s \left[ \begin{array}{c} z^T \\ \overline{e}^T \\ \frac{z^T}{\sigma} \\ \frac{\sigma^T}{\Delta} \end{array} \right] \left[ \begin{array}{c} \Sigma \\ \Gamma \\ \delta \Omega \\ \zeta \overline{e} \end{array} \right] \left[ \begin{array}{c} z^T \\ \overline{e}^T \\ \frac{z^T}{\sigma} \\ \frac{\sigma^T}{\Delta} \end{array} \right] 
$$

where

$$
\Lambda = \left[ \begin{array}{c} \Theta_{ij} \\ \left( \tilde{\Delta}_{ij} + \frac{\tilde{B}_i \tilde{B}_j}{\sigma^2} \right) P_{1} \\ \tilde{\Delta}_{ij} - \frac{\tilde{B}_j \tilde{B}_i}{\sigma^2} \\ 0 \\ 0 \end{array} \right] 
$$

$\Xi$, $\Omega$, and $\Gamma$ are defined in (33). Then, (34) and (42) imply that $\dot{V} < 0$. This completes the proof.

Remark 2: Theorem 2 provides a fuzzy control method to stabilize a nonlinear system with input delay. To apply this method to control a real nonlinear system, an algorithm is presented as follows and a block diagram of observer-based feedback control is shown by Fig. 2.

Step 1) Obtain the mathematical model for the nonlinear plant to be controlled.

Step 2) Set up the fuzzy model for the nonlinear system stated in Step 1 by using the fuzzy modeling method proposed in [17] and [1].

Step 3) Set the fuzzy observer (25) for the T–S fuzzy model proposed in Step 2.

Step 4) Choose the scalars $\sigma > 0$ and $d_1 > 0$.

Step 5) Solve the LMIs (29)–(32) to find feedback gains $K_i$ and observer gains $L_i$.

Step 6) If $K_i$ and $L_i$ do not exist, go back to Step 4 with different scalars $\sigma > 0$ and $d_1 > 0$.

IV. ILLUSTRATIVE EXAMPLES

In this section, some examples are used to illustrate the effectiveness and feasibility of the proposed methods.

Example 1: Consider the truck-trailer systems, which can be described by the following T–S fuzzy system.

Rule 1: IF $\theta(t) = x_2(t) + (\omega^2/2L)x_1(t)$ is about 0, THEN

$$
\dot{x}(t) = A_1 x(t) + B_1 u(t - \tau),
$$

Rule 2: IF $\theta(t) = x_2(t) + (\omega^2/2L)x_1(t)$ is about $\pi$ or $-\pi$, THEN

$$
\dot{x}(t) = A_2 x(t) + B_2 u(t - \tau)
$$

where

$$
A_1 = \begin{bmatrix} -\frac{\omega^2}{L_0^2} & 0 & 0 \\ \frac{L_0^2}{L_0^2} & 0 & 0 \\ \frac{L_0^2}{L_0^2} & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{\omega^2}{L_0^2} \\ \frac{\omega^2}{L_0^2} \end{bmatrix}
$$

$$
A_2 = \begin{bmatrix} -\frac{\omega^2}{L_0^2} & 0 & 0 \\ \frac{L_0^2}{L_0^2} & 0 & 0 \\ \frac{L_0^2}{L_0^2} & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

with $l = 2.8$, $L = 5.5$, $v = -1$, $\omega = 2.0$, $L_0 = 0.5$, and $d = 10\ell_0/\pi$, when $\tau = 0$, the system is the same as one given in [19]. According to [19], we take

$$
h_1 = \left(1 - \frac{1}{1 + \exp(-3(\theta(t) - 0.5\pi))}\right) \\
h_2 = \left(1 + \exp(-3(\theta(t) + 0.5\pi))\right)
$$

$$
h_{l1} = 1 - h_1.
$$
For this T–S fuzzy system, we assume that the model uncertainties are norm-bounded and can be described as $\Delta A_1 = \Delta A_2 = M F(t)E$, $\Delta B_1 = M F(t)E_b$, $M = [0.255 0.255 0.255]^T$, and $E = [0.1 0.1]$, and $E_b = 0.15$. By solving the LMIs (6)–(8) with $d_1 = 0.9$, it can be verified that the maximum allowable value of $\sigma$ is 0.6198 and the corresponding feedback gain matrices are

\[
K_1 = \begin{bmatrix} 4.0540 & -0.4807 & 0.0064 \end{bmatrix},
\]

\[
K_2 = \begin{bmatrix} 4.0540 & -0.4807 & 0.0064 \end{bmatrix}.
\]

For $\tau = 0.5$, the feedback gain matrices are

\[
K_1 = \begin{bmatrix} 4.5009 & -0.9605 & 0.0200 \end{bmatrix},
\]

\[
K_2 = \begin{bmatrix} 4.5009 & -0.9605 & 0.0200 \end{bmatrix}.
\]

The simulation is carried out for $\tau = 0.5$ with the control law of

\[
u = (h_1(t - \tau)K_1 + h_2(t - \tau)K_2)x(t - \tau)
\]

and the initial condition $x(0) = [5 - 3 2]^T$. The simulation results are shown in Figs. 3 and 4. Fig. 3 shows the state responses of the resulting closed-loop system. It is clearly seen that the closed-loop system is asymptotically stable in the presence of an input delay. Fig. 4 shows the control input curve.

Example 2: Consider the following T–S fuzzy system with input delay:

\[
\dot{x} = \sum_{i=1}^{2} h_i (A_i x + B_i u(t - \tau))
\]

\[
y = \sum_{i=1}^{2} h_i C_i x
\]

where

\[
A_1 = \begin{bmatrix} 0.1125 & -0.02 \\ 1 & 0 \end{bmatrix},
\]

\[
A_2 = \begin{bmatrix} 0.1125 & -1.527 \\ 1 & 0 \end{bmatrix},
\]

\[
B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},
\]

\[
C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.
\]

Note that if $\tau = 0$, then this system is the same as one given in [24]. According to [24], it is assumed that $x_2 \in [-1.5, 1.5]$ can be measured, $h_1 = 1 - (x_2^2/2.25)$, and $h_2 = 1 - h_1$. Using Theorem 2 with $d_1 = 105.5$ and $d_2 = 4.5$, it is found that the maximum allowable value of $\sigma$ is 1.35 and the corresponding feedback gains and observer gains are

\[
K_1 = \begin{bmatrix} -0.6455 & -0.6139 \end{bmatrix},
\]

\[
K_2 = \begin{bmatrix} -0.7506 & -0.6430 \end{bmatrix},
\]

\[
L_1 = \begin{bmatrix} 7.2459 \\ 19.7003 \end{bmatrix},
\]

\[
L_2 = \begin{bmatrix} 5.6801 \\ 19.6512 \end{bmatrix}.
\]

The simulation was run under the initial conditions $x(0) = [-1.5 1]^T$, $\dot{x}(0) = [0 0]^T$ for $t \in [-1.35, 0]$. Figs. 5 and 6 show the responses of state and observer error, respectively. Fig. 7 shows the control input action. The simulation results indicate that the proposed control law

\[
u = h_1(t - 1.35)[-1.1057 - 0.5903] \dot{x}(t - 1.35)
\]

\[+ h_2(t - 1.35)[-1.2007 - 0.6433] \dot{x}(t - 1.35)
\]

guarantees the asymptotic stability of the resulting closed-loop system.
Example 3: Consider a nonlinear mass-spring-damper system [9], which is described by the following differential equation:

\[
\ddot{x}(t) = -\dot{x}(t) - 0.01x(t) - 0.1x^3(t) + (1.4387 - 0.13\dot{x}^2(t))u(t),
\]

(43)

According to [9], the operation range for \( x \) is assumed to be within the interval \([-1.5, 1.5]\), and this nonlinear system can be represented by a fuzzy model. The \( il \)th rule is given by

Rule \( i \): IF \( x(t) \) is \( N_{i1} \) and \( \dot{x}(t) \) is \( N_{i2} \) THEN \( \dot{x}(t) = A_i x(t) + B_i u(t) \), \( i = 1, 2, 3, 4 \)

where

\[
A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ -0.01 & -1 \end{bmatrix},
A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ -0.235 & -1 \end{bmatrix},
B_1 = B_3 = \begin{bmatrix} 0 \\ 1.4387 \end{bmatrix},
B_2 = B_4 = \begin{bmatrix} 0 \\ 0.5613 \end{bmatrix}
\]

and the membership functions of \( N_{ik} \), \( i = 1, 2, 3, 4 \), and \( k = 1, 2 \), are defined as follows:

\[
N_{11}(x(t)) = N_{21}(x(t)) = 1 - \frac{x^2(t)}{2.25},
N_{31}(x(t)) = N_{41}(x(t)) = \frac{x^2(t)}{2.25},
N_{12}(\dot{x}(t)) = N_{22}(\dot{x}(t)) = 1 - \frac{\dot{x}^2(t)}{6.75},
N_{32}(\dot{x}(t)) = N_{42}(\dot{x}(t)) = \frac{\dot{x}^2(t)}{6.7}.
\]

Without considering the impact of input delay, the following fuzzy control law is presented in [9] to control the original nonlinear system (43)

\[
u = \sum_{i=1}^{2} h_i K_{i} x
\]

\[
K_1 = \begin{bmatrix} -2.7732 & -2.0852 \\ -6.7076 & -5.3447 \end{bmatrix}
\]

(44)

Under the action of the above control law, the simulation is carried out with the initial conditions \( x(0) = [1.5, -1]^T \). It was found that the closed-loop stability is guaranteed by the control law (44) for any input delay \( \tau < 0.5 \). But when the input delay is larger than 0.5, the closed-loop system becomes unstable. Fig. 8 shows the closed-loop response under the action of the fuzzy controller suggested in [9] for the case of \( \tau = 0.5 \).

However, Theorem 1 can be used to design a fuzzy controller to attenuate the influence of input delay. The following uncertainties are added to the system to check if the proposed controller is able to stabilize the uncertain system:

\[
MF(t)E_i = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix} F(t) [0.1, 0.5], \quad i = 1, 2, 3, 4.
\]

According to Theorem 1, a fuzzy controller is designed for the system with input delay and uncertainties. By solving LMIs
\[ K_1 = \begin{bmatrix} -0.1249 & -0.1035 \\ -0.0850 & -0.0738 \\ -0.1300 & -0.1129 \\ -0.0819 & -0.0648 \end{bmatrix}, \]

The fuzzy control law \( u = \sum_{i=1}^{4} h_i(\tau)K_{ix} \) is used to control the original nonlinear system. Under this fuzzy control action, the closed-loop asymptotical stability is guaranteed if the input delay \( \tau \) is not larger than 3.5160. For simplicity, in the simulation, the input delay is considered as a constant given by \( \tau(t) = 3.5160 \). The simulation results are shown by Figs. 9 and 10. Fig. 9 shows the state response for the case \( \tau = 3.5160 \), and Fig. 10 shows the control curve.

Example 4: Consider the following inverted pendulum system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{g \sin(x_1) - amx_2^2 \sin(2x_1)/2 - a \cos(x_1)u}{a/3 - am \cos^2(x_1)} 
\end{align*}
\tag{45}
\]

where \( x_1 \) denotes the angle of the pendulum from the vertical, \( x_2 \) stands for the angular velocity, \( g = 9.8 \text{ m/s}^2 \) is the gravity constant, \( m \) is the mass of the pendulum, \( M \) is the mass of the cart, \( 2l \) is the length of the pendulum, and \( u \) is the force applied to the cart. The system parameters are chosen as

\[
m = 2.0 \text{ kg}, \quad M = 8.0 \text{ kg} \\
2l = 1.0 \text{ m}, \quad a = 1/(m + M).
\]

Choose \( x_1 \) as the system output, then according to [13], the nonlinear system (45) can be presented by the following T–S fuzzy model:

\[
x = \sum_{i=1}^{2} h_i(A_i x + B_i u), \quad y = \sum_{i=1}^{2} h_i C_i x
\]

where

\[
A_1 = \begin{bmatrix} 0 & 1 \\ 17.294 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix}, \\
C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
h_1 = \left(1 - \frac{1}{1 + \exp\left(-7(x_1 - \pi/4)\right)}\right), \\
h_2 = \left(\frac{1}{1 + \exp\left(-7(x_1 + \pi/4)\right)}\right) - 1.
\]
By using Theorem 2 with $d_1 = d_2 = 1.5$, the feedback gains and observer gains are obtained as

$$K_1 = \begin{bmatrix} 191.4511 \\ 49.1305 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 191.4511 \\ 49.1305 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 2.4955 \\ 17.5279 \end{bmatrix}^T,$$

$$L_2 = \begin{bmatrix} 2.5873 \\ 12.8505 \end{bmatrix}^T,$$

and the maximum allowable value of input delay is $\tau = 0.2105(\text{s})$. The simulation is run with the initial conditions $x(0) = [1.2 \ 2]$, $\dot{x}(0) = [0 \ 0]$, and $\tau = 0.2105$. Figs. 12 and 13 show the simulation results.

For the case without the input delay, by using the method proposed in [22], the control gains and observer gains are given as follows:

$$K_1 = \begin{bmatrix} 409.7513 \\ 99.2353 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 512.9116 \\ 121.7956 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.7283 \\ 18.3000 \end{bmatrix}^T,$$

$$L_2 = \begin{bmatrix} 0.6242 \\ 13.6408 \end{bmatrix}^T.$$

The simulation results show that with the control strategy suggested by [22] the closed-loop system is asymptotically stable if $\tau < 0.05$ and becomes unstable when $\tau > 0.05$, as shown in Fig. 14.

V. CONCLUSION

In this paper, we have developed a new fuzzy controller design methodology for T–S fuzzy systems with bounded and time-varying input delay. The main contribution lies in that some sufficient conditions of delay-dependent stabilization are proposed in terms of LMIs. The effect of input delay has been considered when the controller is designed. The suggested fuzzy controller guarantees the asymptotic stability of the resulting closed-loop system not only when input delay is free but also when the input delay is less than an allowable upper bound. Some examples are used to give the comparison between our results and the existing methods. The simulation results show that compared with the existing methods, the controller proposed in this paper is more robust against the input delay. It should be pointed out that the uncertainties are not considered for the problem of the observer-based stabilization. It is found that it is a challenge to derive a separate design principle for observer-based stabilization of uncertain T–S fuzzy systems.

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